Algorithms for two-time scales stochastic optimization

with applications to long term management of energy storage

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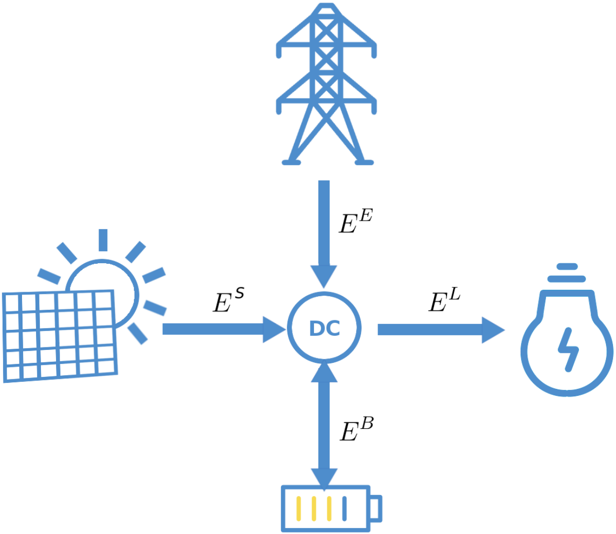
Outline

Microgrid & stochastic optimization at Efficacity



Publications: from small problems to large problems

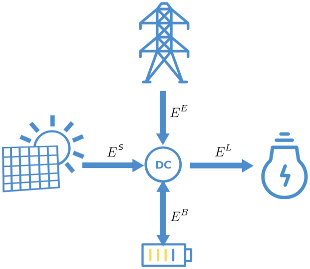
- **Stochastic optimal control of a domestic microgrid equipped with solar panel and battery.** Pacaud, F., Carpentier, P., Chancelier, J.P., De Lara, M.
- **Stochastic Optimization of Braking Energy Storage and Ventilation in a Subway Station**, T. Rigaut, P. Carpentier, J-Ph. Chancelier, M. De Lara, J. Waeytens.
- **Stochastic decomposition applied to large-scale hydro valleys management.**, Pacaud, F., Carpentier, P., Chancelier, J.P. and Leclere, V.
- **Algorithms for two-time scales stochastic optimization with applications to long term management of energy storage**, Rigaut, T., Carpentier, P., Chancelier, J.P. and De Lara, M.

Outline

Energy system : a house with solar panels

All the equipment exchange electricity though a DC grid.

$$\boldsymbol{E}^{E}_{d,m+1} + \boldsymbol{E}^{S}_{d,m+1} = \boldsymbol{E}^{B}_{d,m} + \boldsymbol{E}^{L}_{d,m+1}$$



DC microgrid to be managed

- *DC*: Very small storage on a really fast time scale
- E^L: Electrical load, or demand, that is uncertain
- E^S: Solar panels, uncertain renewable electricity
- E^E: Connection to the national grid (recourse)
- E^B: Electrical storage (charge/discharge)

A Two time scales decision process

- $M \in \mathbb{N}^*$ the number of minutes in a day,
- $D \in \mathbb{N}^*$, the number of days taken into account.
- Decisions:
  - Battery charge/discharge every minutes $m \in \{0, \dots, M\}$ of every day $d \in \{0, \dots, D\}$,
  - Renewal of the battery or not every day $d \in \{0, \dots, D+1\}$.
- Notations:
  - Two time indexes $z_{d,m}$: z changes every minutes m of everyday d
  - Single index z_d: z changes only every day
  - $(d, m) \in \mathbb{T}$ with

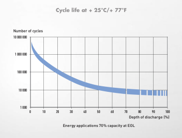
$$\mathbb{T} = \{0, \dots, D\} \times \{0, \dots, M\} \cup \{(D+1, 0)\} ,$$

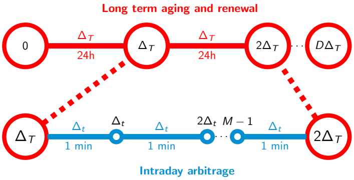
equipped with the *lexicographical order*

$$(d, m) < (d', m') \iff (d < d') \vee \big(d = d' \wedge m < m'\big) .$$

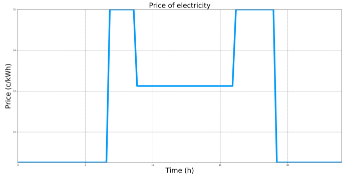
Two time scales

- Long term economic profitability
- Horizon: 10 years (d: step is 1 day)
- Storage aging target every day





- Energy intraday arbitrage
- Horizon: 24h (m: step is 1 min)
- charge/discharge



Uncertainties

- Fast scale:
  - $\boldsymbol{E}^S_{d,m}$: the solar production in kWh,
  - $\boldsymbol{E}^L_{d,m}$: the electrical demand (load) in kWh.
- Slow scale:
  - $\boldsymbol{P}^b_d$: the price of a battery replacement in $\$/kWh$.
- Gather uncertainties as the sequence $\left\{\boldsymbol{W}_{d,m}\right\}_{(d,m)\in\mathbb{T}}$:

$$\boldsymbol{W}_{d,m} = \begin{pmatrix} \boldsymbol{E}^S_{d,m} \\ \boldsymbol{E}^L_{d,m} \end{pmatrix} \text{ and } \boldsymbol{W}_{d,M} = \begin{pmatrix} \boldsymbol{E}^S_{d,M} \\ \boldsymbol{E}^L_{d,M} \\ \boldsymbol{P}^b_d \end{pmatrix}$$

- Information at time (d, m): past observations of noises

$$\mathcal{F}_{d,m} = \sigma\left(\boldsymbol{W}_{d',m'}; (d', m') \leq (d, m)\right)$$

Non anticipative decisions

- Physical decision variables
  - Fast scale:
    - $\boldsymbol{E}^E_{d,m}$: the national grid consumption in kWh;
    - $\boldsymbol{E}^B_{d,m}$: the battery charge (≥ 0) or discharge (≤ 0) in kWh.
  - Slow scale:
    - $\boldsymbol{R}_d$: the size of the new battery in kWh.
- Mathematical decision variables
  - $\boldsymbol{E}^E_{d,m}$ is supposed to be imposed by non modelized dynamics:

$$\boldsymbol{E}^E_{d,m+1} = \boldsymbol{E}^B_{d,m} + \boldsymbol{E}^L_{d,m+1} - \boldsymbol{E}^S_{d,m+1}$$

  - Controls are grouped as:

$$\boldsymbol{U}_{d,m} = \left(\boldsymbol{E}^B_{d,m}\right) \text{ and } \boldsymbol{U}_{d,M} = \left(\boldsymbol{R}_d\right)$$

Charge/discharge impacts battery state of charge and age

- Fast state dynamics
  - $\boldsymbol{C}_d$: capacity of the battery
  - $\boldsymbol{B}_{d,m}$: state of charge of the battery

$$\boldsymbol{B}_{d,m+1} = \boldsymbol{B}_{d,m} - \frac{1}{\rho_d}\boldsymbol{E}^{B-}_{d,m} + \frac{1}{\rho_d}\rho_c\boldsymbol{E}^{B+}_{d,m}$$
$$\text{s.t. } \underline{B} \times \boldsymbol{C}_d \times \leq \boldsymbol{B}_{d,m} \leq \overline{B} \times \boldsymbol{C}_d$$

  - $\boldsymbol{H}_{d,m}$: remaining amount of exchangeable energy (health measure)

$$\boldsymbol{H}_{d,m+1} = \boldsymbol{H}_{d,m} - \frac{1}{\rho_d}\boldsymbol{E}^{B-}_{d,m} - \rho_c\boldsymbol{E}^{B+}_{d,m}$$
$$\text{s.t. } 0 \leq \boldsymbol{H}_{d,m}$$

(max number of cycles gives initial value as) $2 \times N_c(\boldsymbol{C}_d) \times \boldsymbol{C}_d$

Battery renewal impacts state dynamics

- Physical fast state dynamics
  - Capacity

$$\boldsymbol{C}_{d+1} = \begin{cases} \boldsymbol{R}_d \ , & \text{if } \boldsymbol{R}_d > 0 \ , \\ \boldsymbol{C}_d \ , & \text{otherwise} \ . \end{cases}$$

  - Charge: new battery is assumed empty

$$\boldsymbol{B}_{d+1,0} = \begin{cases} \underline{B} \times \boldsymbol{R}_d \ , & \text{if } \boldsymbol{R}_d > 0 \ , \\ \boldsymbol{B}_{d,M} \ , & \text{otherwise} \ , \end{cases}$$

  - Exchangeable energy (new battery has a renewed health)

$$\boldsymbol{H}_{d+1,0} = \begin{cases} 2 \times N_c(\boldsymbol{R}_d) \times \boldsymbol{R}_d \ , & \text{if } \boldsymbol{R}_d > 0 \ , \\ \boldsymbol{H}_{d,M} \ , & \text{otherwise} \ . \end{cases}$$

- Mathematical daily state dynamics $\boldsymbol{X}_d$:

$$\boldsymbol{X}_d = \begin{pmatrix} \boldsymbol{C}_d \\ \boldsymbol{B}_{d,0} \\ \boldsymbol{H}_{d,0} \end{pmatrix} \text{ and } \boldsymbol{X}_{d+1} = f_d\big(\boldsymbol{X}_d, \boldsymbol{U}_{d,0:M}\big)$$

Stochastic optimization problem

Objective to be minimized: discounted sum of expenses, that is battery renewals cost and national grid energy consumption cost

$$\mathbb{E}\Big[\sum_{d=0}^{D}\gamma_d\Big(\underbrace{\boldsymbol{P}^b_d \times \boldsymbol{R}_d}_{\text{battery renewal}} + \sum_{m=0}^{M-1}\underbrace{p^e_{d,m}}_{\textit{price}}\times\big(\underbrace{\boldsymbol{E}^B_{d,m} + \boldsymbol{E}^L_{d,m+1} - \boldsymbol{E}^S_{d,m+1}}_{E^E_{d,m+1}\text{(nat. grid energy consumption)}}\big)\Big)\Big]$$

Gathering all the above equations, we obtain:

$$\begin{aligned}
V(x) = \min_{\boldsymbol{X}_{0:D+1},\,\boldsymbol{U}_{0:D}} \ & \mathbb{E}\Big[\sum_{d=0}^{D} L_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + K(\boldsymbol{X}_{D+1})\Big] , \\
\text{s.t } & \boldsymbol{X}_{d+1} = f_d(\boldsymbol{X}_d, \boldsymbol{U}_d, \boldsymbol{W}_d) , \\
& \boldsymbol{U}_d = (\boldsymbol{U}_{d,0}, \dots, \boldsymbol{U}_{d,m}, \dots, \boldsymbol{U}_{d,M}) , \\
& \boldsymbol{W}_d = (\boldsymbol{W}_{d,0}, \dots, \boldsymbol{W}_{d,m}, \dots, \boldsymbol{W}_{d,M}) , \\
& \sigma(\boldsymbol{U}_{d,m}) \subset \sigma\big(\boldsymbol{W}_{d',m'}; (d',m') \leq (d,m)\big) \\
& \boldsymbol{X}_0 = x ,
\end{aligned}$$

Outline

Time blocks dynamic programming

Independance Assumption

The sequence $\{\boldsymbol{W}_d\}_{d=0,\dots,D}$ is a sequence of independent random variables ($\boldsymbol{W}_d = (\boldsymbol{W}_{d,0}, \dots, \boldsymbol{W}_{d,M})$)

Sequence of daily value functions, defined by backward induction as follows. At time $D+1$, we set $V_{D+1} = K$ and then

$$V_d(x) = \min_{\boldsymbol{X}_{d+1}, \boldsymbol{U}_d} \mathbb{E}\left[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) + V_{d+1}(\boldsymbol{X}_{d+1})\right]$$

$$\text{s.t } \boldsymbol{X}_{d+1} = f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)$$

$$\sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Proposition [?]

Under Independance Assumption $V_0 = V$

Time blocks decomposition

The target intraday problem (min min problem)

$$\mathcal{P}_{(d,=)}\left[x_d, \boldsymbol{X}_{d+1}\right] \begin{cases} \min\limits_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d)\right] \\ \quad \text{s.t } f_d(x, \boldsymbol{U}_d, \boldsymbol{W}_d) = \boldsymbol{X}_{d+1} \\ \qquad \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m}) \end{cases}$$

Proposition

Under Independence Assumption, V_d satisfy: $V_{D+1} = K$

$$V_d(x) = \min_{\boldsymbol{X} \in L^0(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{X}_{d+1})} \Big(\phi_{(d,=)}\big(x, \boldsymbol{X}\big) + \mathbb{E}\big[V_{d+1}(\boldsymbol{X})\big]\Big) ,$$

$$\text{s.t } \sigma(\boldsymbol{X}) \subset \sigma(\boldsymbol{W}_d) .$$

where $\phi_{(d,=)}\big(x_d, \boldsymbol{X}_{d+1}\big)$ is the value of $\mathcal{P}_{(d,=)}\big[x_d, \boldsymbol{X}_{d+1}\big]$

Relaxed Time blocks decomposition

A relaxed target intraday problem (min min problem)

$$\mathcal{P}_{(d,\geq)}\left[x_d, \boldsymbol{X}_{d+1}\right] \begin{cases} \min_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d)\right] \\ \quad \text{s.t } f_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d) \geq \boldsymbol{X}_{d+1} \\ \qquad \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m}) \end{cases}$$

A relaxed Bellman value function $V_{(d,\geq)}$

$V_{(d,\geq)}$ satisfy: $V_{(D+1)} = K$

$$V_{(d,\geq)}(x) = \min_{\boldsymbol{X} \in L^0(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{X}_{d+1})} \Big(\phi_{(d,\geq)}(x, \boldsymbol{X}) + \mathbb{E}\left[V_{(d+1,\geq)}(\boldsymbol{X})\right]\Big) ,$$

$$\text{s.t } \sigma(\boldsymbol{X}) \subset \sigma(\boldsymbol{W}_d) .$$

where $\phi_{(d,\geq)}(x_d, \boldsymbol{X}_{d+1})$ is the value of $\mathcal{P}_{(d,\geq)}\left[x_d, \boldsymbol{X}_{d+1}\right]$

Relaxed Time blocks decomposition

Assumption

The final cost K is a non increasing mapping and that for all $d \in \{0, \dots, D\}$, the dynamics f_d are non decreasing over their first argument and that the instantaneous costs L_d are non increasing over their first argument.

Proposition

$V_{(d,\geq)} \leq V_d$ and under above Assumption $V_d = V_{(d,\geq)} \leq V_{(d,\geq,\mathbb{X}_{d+1})}$

$$V_{(d,\geq,\mathbb{X}_{d+1})}(x) = \min_{X \in \mathbb{X}_{d+1}} \Big(\phi_{(d,\geq)}\big(x, X\big) + V_{(d+1,\geq,\mathbb{X}_{d+1})}(X) \Big)$$

Main numerical efforts compute $\phi_{(d,\geq)}\big(\cdot,\cdot\big)$

- May depend on $x - x'$, $(\phi_{(d,\geq)}\big(x - x'\big))$. Subset of days.
- SP methods, Progressive hedging methods
- Parallelism (on variable d, on states (x, x'))

Adaptative weight algorithm

Dualized intraday problems ψ_d, $(x_d, \boldsymbol{\lambda}_{d+1}) \in \mathbb{X}_d \times L^0(\Omega, \mathcal{F}, \mathbb{P}; \Lambda_{d+1})$

$$\psi_d(x_d, \boldsymbol{\lambda}_{d+1}) = \min_{\boldsymbol{U}_d} \mathbb{E}\left[L_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d) + \langle \boldsymbol{\lambda}_{d+1}, f_d(x_d, \boldsymbol{U}_d, \boldsymbol{W}_d)\rangle\right]$$
$$\text{s.t } \sigma(\boldsymbol{U}_{d,m}) \subset \sigma(\boldsymbol{W}_{d,0:m})$$

Adaptative daily value function $\underline{V}_d$

$\underline{V}_d$ satisfy: $\underline{V}_{D+} = K$

$$\underline{V}_d(x_d) = \sup_{\boldsymbol{\lambda}_{d+1} \in \Lambda_{d+1}} \psi_d(x_d, \boldsymbol{\Lambda}_{d+1}) - \underline{V}^*_{d+1}(\boldsymbol{\lambda}_{d+1}) \,,$$
$$\text{s.t } \sigma(\boldsymbol{\lambda}_{d+1}) \subset \sigma(\boldsymbol{X}_{d+1}) \,,$$

where $\underline{V}^*_{d+1}$ is the Fenchel transform of the function $\underline{V}_{d+1}$.

Adaptative weight algorithm

Lemma

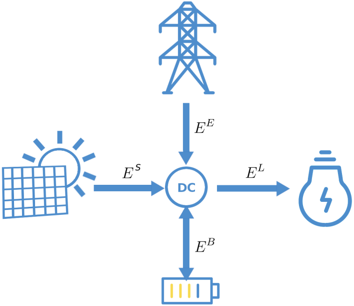
$\underline{V}_d \leq V_d$. Assume that K is convex non increasing and that the dynamics f_d are non decreasing over their first argument and linear and that the instantaneous costs L_d are non increasing over their first argument and convex. If moreover $ri\Big(dom(\psi_d(x_d, \cdot)) - dom(\mathbb{E}V_{d+1}(\cdot))\Big) \neq \emptyset$. Then, the value functions V_d are non increasing and we have the equality $V_d = \underline{V}_d$

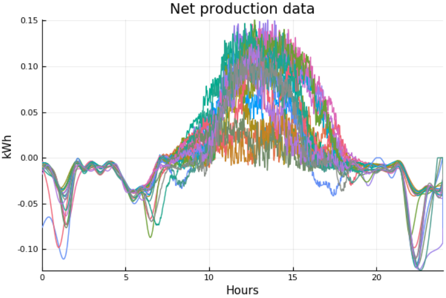
- Computationally costly to compute the function ψ_d for every $d \in \{0, \dots, D\}$, initial state $x_d \in \mathbb{X}_d$ and particularly stochastic weights $\boldsymbol{\lambda} \in L^0(\Omega, \mathcal{F}, \mathbb{P}; \Lambda_{d+1})$.
- (Heuristic) Restrict the computation to deterministic weights in Λ_{d+1}.

Outline

A house with solar panels and a battery

- Solar radiation measurements from Zambia[1] converted into solar panels (12kWc) production with PVLIB[2]
- Load data from a customer in Australia[3]
- We want to minimize the electricity bill of the house!





[1] energydata.info/en/dataset/zambia-solar-radiation-measurement-data-2015-2017
[2] github.com/pvlib/pvlib-python
[3] www.ausgrid.com.au/datatoshare

Application 1: comparison on a simple aging problem

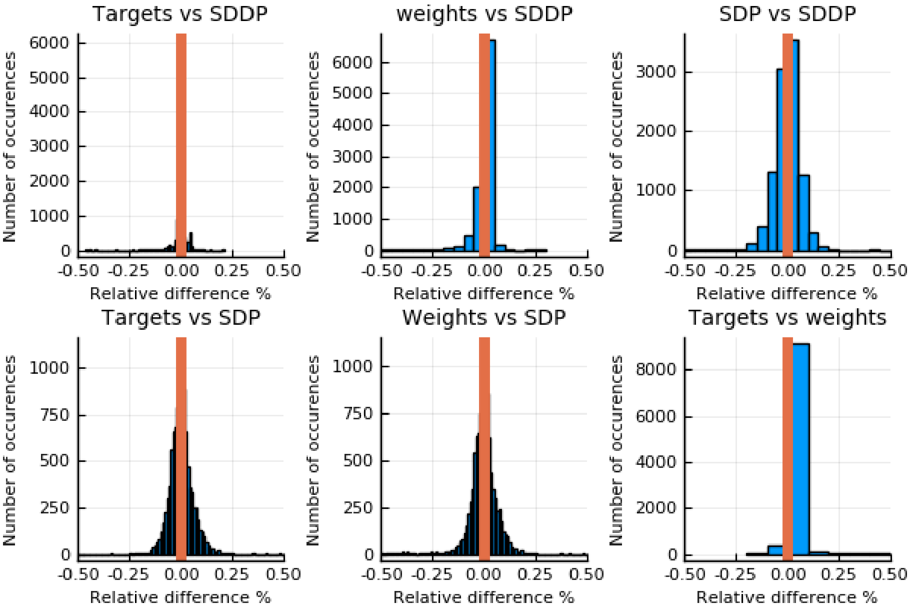
Instance :

- 5 days, 7200 minutes
- 13 kWh battery, 100 kWh of exchangeable energy
- No battery renewal!
- We control state of charge and aging every minutes

Algorithms :

- Straightforward stochastic dynamic programming
- Daily time blocks decomposition with targets
- Daily time decomposition with weigths
- Straightforward stochastic dual dynamic programming

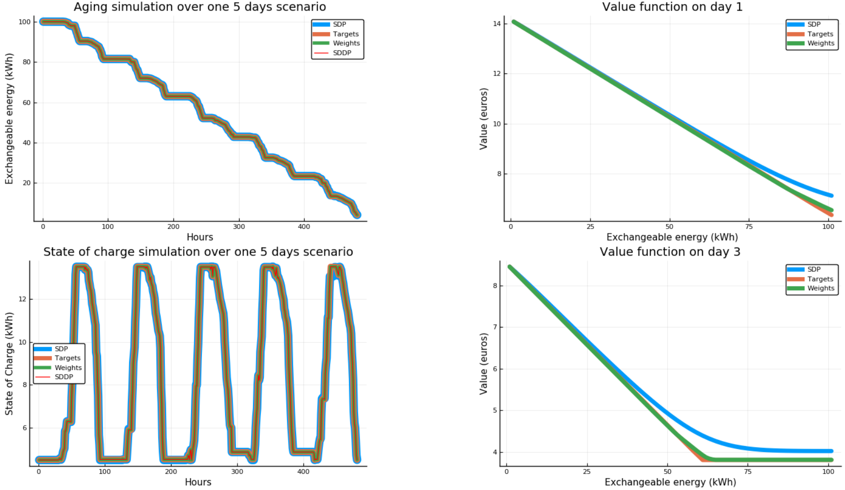
In-sample assessment



Computation times and convergence

	SDP	Targets	Weights	SDDP
Intraday (SDDP)	n.a	14 sec	51×14 sec	n.a
Daily values	n.a	0.10 sec	0.15 sec	n.a
Minute values	22.5 min	5×14 sec	5×4.5 min	3.6 min
Convergence	0.91 %	0.31 %	0.32 %	0.90 %

Simulations and value functions comparison



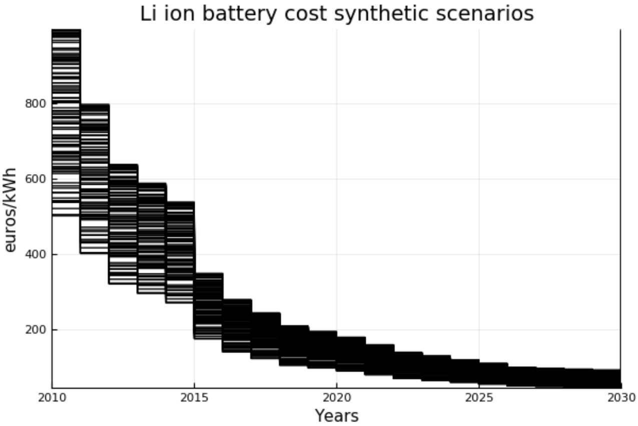
Application 2: A case with renewal

Instance :

- 20 years, 10512000 minutes
- Battery capacity between 0 and 20 kWh
- Initial health : $2 \times N_{cycles} \times capacity$
- Renewal possible everyday
- We control state of charge and aging every minutes
- Yearly discount rate : 0.96

Synthetic price of batteries

- Batteries cost stochastic model: synthetic scenarios that approximately coincide with market forecasts[4]

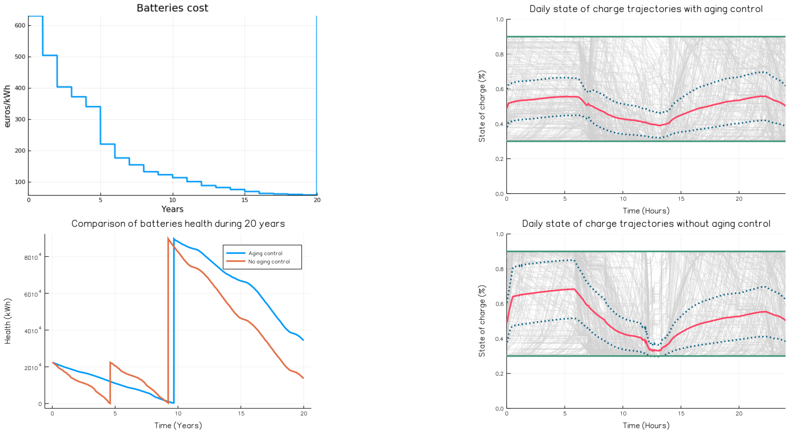


[4]Bloomberg forecasts: data.bloomberglp.com/bnef/sites/14/2017/07/BNEF-Lithium-ion-battery-costs-and-market.pdf

1 simulation over 20 years: it pays to control aging!

Reference: optimal battery and aging control

- No aging control: +8% of expenses over 20 years,
- No battery: +10% of expenses over 20 years.



Application 3: Optimal sizing of a battery